Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

DIOPHANTINE ANALYSIS.

121. Proposed by **L.E. DICKSON, Ph.D., Assisstant Professor in Mathematics, The University of Chicago.**

Find a formula for the solutions of $x^2+y^2\equiv 1$ (mod$p$) valid in all cases $p>2$.

I. Solution by the **PROPOSER.**

Let $e=\pm 1$ according as $p=4l\pm 1$, *i. e.*, according as -1 is or is not the square of an integer i modulo p. Then the desired formula is

$$x\equiv\tfrac{1}{2}\left(z+\frac{1}{z}\right),\quad y\equiv\frac{-i}{2}\left(z-\frac{1}{z}\right),\text{ where } z^{p-e}\equiv 1.$$

The proof is evident when i is an integer. In the contrary case, $i^2=-1$ defines the $GF[p^2]$ in which $z^{p+1}=1$ has $p+1$ solutions z. Let $x+iy=z$, whence $x-iy\equiv x+i^py\equiv z^p$, $(x+iy)(x-iy)=z^{p+1}=1$.

II. Solution by **G. B. M. ZERR, A. M., Ph. D.**, Parsons, W. Va.

This is equivalent to finding a general formula for $x^2+y^2-pz=1$.

Let $x=mz$, $y=nz$. $\therefore (m^2+n^2)z^2-pz=1$. $\therefore z=\dfrac{p\pm\sqrt{[p^2+4(m^2+n^2)]}}{2(m^2+n^2)}$.

Let $p=r^2+s^2-t^2$, $m=rt$, $n=st$. $\therefore z=\dfrac{r^2+s^2-t^2\pm(r^2+s^2+t^2)}{2t^2(r^2+s^2)}$.

$$\therefore z=\frac{1}{t^2}\text{ or }-\frac{1}{r^2+s^2}.\qquad \therefore x=mz=\frac{r}{t}\text{ or }-\frac{rt}{r^2+s^2}.$$

$$y=nz=\frac{s}{t}\text{ or }-\frac{st}{r^2+s^2}.\qquad \therefore\frac{r^2}{t^2}+\frac{s^2}{t^2}-\frac{r^2+s^2-t^2}{t^2}=1\,\ldots\ldots\ldots(1),$$

$$\text{or }\frac{r^2t^2}{(r^2+s^2)^2}+\frac{s^2t^2}{(r^2+s^2)^2}+\frac{r^2+s^2-t^2}{r^2+s^2}=1\,\ldots\ldots\,(2).$$

Both (1) and (2) are general formulae satisfying the conditions.

GROUP THEORY.

4. Proposed by **SAUL EPSTEEN**, Chicago, Ill.

Find the four-parameter continuous group which leaves invariant $x_2^2-x_1x_3=0$.

Solution by the **PROPOSER.**

Writing

$$\begin{aligned}x_1'&=a_{11}x_1+a_{12}x_2+a_{13}x_2\\x_2'&=a_{21}x_1+a_{22}x_2+a_{23}x_3\\x_3'&=a_{31}x_1+a_{32}x_2+a_{33}x_3\end{aligned}$$

and expressing the condition that $x_2'^2-x_1'x_3'=x_2^2-x_1x_3$ we obtain six equations for the nine quantities a_{ij} $(i, j=1, 2, 3)$. Expressed parametrically,

$$\begin{aligned}&a_{11}=a^2,\quad a_{12}=2ab,\quad a_{13}=b^2,\\&a_{21}=ac,\quad a_{22}=(ad+bc),\quad a_{23}=bd,\\&a_{31}=c^2,\quad a_{32}=2cd,\quad a_{33}=d^2,\end{aligned}$$